OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4761

Mechanics 1

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use g = 9.8.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (36 marks)

1 A particle travels along a straight line. Its *acceleration* during the time interval $0 \leqslant t \leqslant 8$ is given by the acceleration–time graph in Fig. 1.

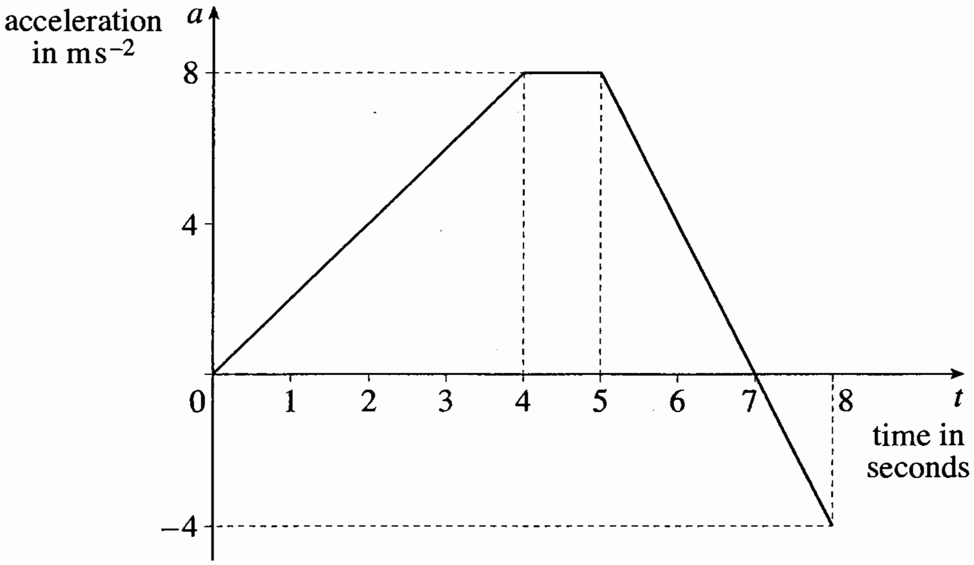


Fig. 1

(i) Write down the acceleration of the particle when $t = 4$. Given that the particle starts from rest, find its speed when $t = 4$. [2]

(ii) Write down an expression in terms of t for the acceleration, $a\,\mathrm{m\,s^{-2}}$, of the particle in the time interval $0 \leqslant t \leqslant 4$. [1]

(iii) Without calculation, state the time at which the *speed* of the particle is greatest. Give a reason for your answer. [2]

(iv) Calculate the change in speed of the particle from $t = 5$ to $t = 8$, indicating whether this is an increase or a decrease. [3]

2 A particle moves along the x-axis with velocity, $v\,\mathrm{m\,s^{-1}}$, at time t given by

$$v = 24t - 6t^2.$$

The positive direction is in the sense of x increasing.

(i) Find an expression for the acceleration of the particle at time t. [2]

(ii) Find the times, t_1 and t_2, at which the particle has zero speed. [2]

(iii) Find the distance travelled between the times t_1 and t_2. [4]

3 A particle rests on a smooth, horizontal plane. Horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ lie in this plane. The particle is in equilibrium under the action of the three forces $(-3\mathbf{i} + 4\mathbf{j})$ N and $(21\mathbf{i} - 7\mathbf{j})$ N and $\mathbf{R}$ N.

(i) Write down an expression for $\mathbf{R}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [2]

(ii) Find the magnitude of $\mathbf{R}$ and the angle between $\mathbf{R}$ and the $\mathbf{i}$ direction. [4]

4 A block of mass 4 kg is in equilibrium on a rough plane inclined at 60° to the horizontal, as shown in Fig. 4. A frictional force of 10 N acts up the plane and a vertical string AB attached to the block is in tension.

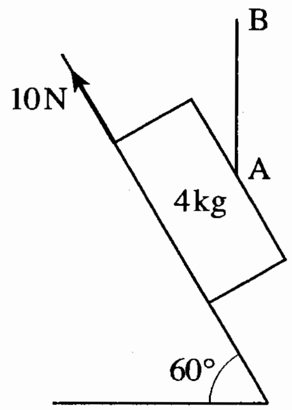


Fig. 4

(i) Draw a diagram showing the four forces acting on the block. [1]

(ii) By considering the components of the forces parallel to the slope, calculate the tension in the string. [3]

(iii) Calculate the normal reaction of the plane on the block. [3]

5 The position vector of a particle at time t is given by

$$\mathbf{r} = \tfrac{1}{2}t\mathbf{i} + (t^2 - 1)\mathbf{j},$$

referred to an origin O where $\mathbf{i}$ and $\mathbf{j}$ are the standard unit vectors in the directions of the cartesian axes Ox and Oy respectively.

(i) Write down the value of t for which the x-coordinate of the position of the particle is 2. Find the y-coordinate at this time. [2]

(ii) Show that the cartesian equation of the path of the particle is $y = 4x^2 - 1$. [2]

(iii) Find the coordinates of the point where the particle is moving at 45° to both Ox and Oy. [3]

Section B (36 marks)

6 A car of mass 1000 kg is travelling along a straight, level road.

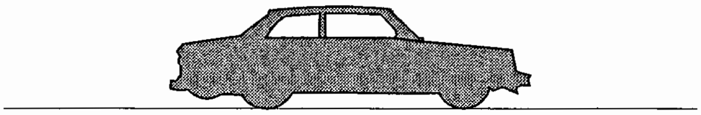


Fig. 6.1

(i) Calculate the acceleration of the car when a resultant force of 2000 N acts on it in the direction of its motion.

How long does it take the car to increase its speed from $5 \mathrm{~m\,s^{-1}}$ to $12.5 \mathrm{~m\,s^{-1}}$? [3]

The car has an acceleration of $1.4 \mathrm{~m\,s^{-2}}$ when there is a driving force of 2000 N.

(ii) Show that the resistance to motion of the car is 600 N. [2]

A trailer is now atached to the car, as shown in Fig. 6.2. The car still has a driving force of 2000 N and resistance to motion of 600 N. The trailer has a mass of 800 kg. The tow-bar connecting the car and the trailer is light and horizontal. The car and trailer are accelerating at $0.7 \mathrm{~m\,s^{-2}}$.

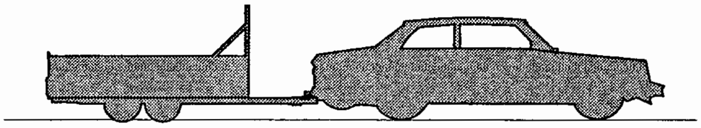


Fig. 6.2

(iii) Show that the resistance to the motion of the trailer is 140 N. [3]

(iv) Calculate the force in the tow-bar. [3]

The driving force is now removed and a braking force of 610 N is applied to the car. All the resistances to motion remain as before. The trailer has no brakes.

(v) Calculate the new acceleration. Calculate also the force in the tow-bar, stating whether it is a tension or a thrust (compression). [6]

7 In this question take the value of g to be $10\,\mathrm{m\,s^{-2}}$.

A particle A is projected over horizontal ground from a point P which is 9 m above a point O on the ground. The initial velocity has horizontal and vertical components of $10\,\mathrm{m\,s^{-1}}$ and $12\,\mathrm{m\,s^{-1}}$ respectively, as shown in Fig. 7. The trajectory of the particle meets the ground at X. Air resistance may be neglected.

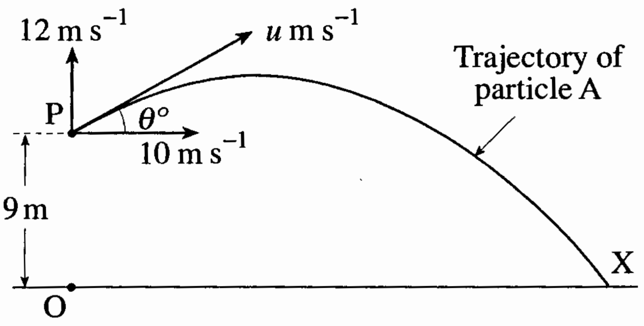


Fig. 7

(i) Calculate the speed of projection $u\,\mathrm{m\,s^{-1}}$ and the angle of projection $\theta°$. [3]

(ii) Show that, t seconds after projection, the height of particle A above the ground is $9 + 12t - 5t^2$. Write down an expression in terms of t for the horizontal distance of the particle from O at this time. [4]

(iii) Calculate the maximum height of particle A above the point of projection. [2]

(iv) Calculate the distance OX. [4]

A second particle, B, is projected from O with speed $20\,\mathrm{m\,s^{-1}}$ at 60° to the horizontal. The trajectories of A and B are in the same vertical plane. Particles A and B are projected at the same time.

(v) Show that the horizontal displacements of A and B are always equal. [2]

(vi) Show that, t seconds after projection, the height of particle B above the ground is $10\sqrt{3}t - 5t^2$. [1]

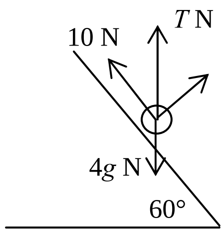
(vii) Show that the particles collide 1.7 seconds after projection (correct to two significant figures). [3]

Mark Scheme 4761
June 2005

Q 1		mark		Sub
(i)	Acceleration is 8 m s^{-2} speed is $0 + 0.5 \times 4 \times 8 = 16$ m s^{-1}	B1 B1		2
(ii)	$a = 2t$	B1		1
(iii)	$t = 7$ $a > 0$ for $t < 7$ and $a < 0$ for $t > 7$	B1 E1	 Full reason required	2
(iv)	Area under graph $0.5 \times 2 \times 8 - 0.5 \times 1 \times 4 = 6$ so 6 m s^{-1} Increase	M1 B1 E1	Both areas under graph attempted. Accept both positive areas. If 2×3 seen accept ONLY IF reference to average accn has been made. Award for $v = -2t^2 + 28t + c$ seen or 24 and 30 seen Award if 6 seen. Accept '24 to 30'. This must be clear. Mark dept. on award of M1	3
	total	8		

Q 2		mark		Sub
(i)	$a = 24 - 12t$	M1 A1	Differentiate cao	2
(ii)	Need $24t - 6t^2 = 0$ $t = 0, 4$	M1 A1	Equate $v = 0$ and attempt to factorise (or solve). Award for one root found. Both. cao.	2
(iii)	$s = \int_0^4 \left(24t - 6t^2\right) \mathrm{d}t$ $= \left[12t^2 - 2t^3\right]_0^4$ $\left(12 \times 16 - 2 \times 64\right) - 0$ $= 64$ m	M1 A1 M1 A1	Attempt to integrate. No limits required. Either term correct. No limits required Sub $t = 4$ in integral. Accept no bottom limit substituted or arb const assumed 0. Accept reversed limits. FT **their** limits. cao. Award if seen. [If trapezium rule used. M1 At least 4 strips: M1 enough strips for 3 s. f. A1 (dep on 2nd M1) One strip area correct: A1 cao]	4
	total	8		

Q 3		mark		Sub
(i)	$\mathbf{R} + \begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 21 \\ -7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	M1	Sum to zero	
	$\mathbf{R} = \begin{pmatrix} -18 \\ 3 \end{pmatrix}$	A1	Award if seen here or in (ii) or used in (ii).	
			[SC1for $\begin{pmatrix} 18 \\ -3 \end{pmatrix}$]	2
(ii)	$\lvert\ \mathbf{R}\ \rvert = \sqrt{18^2 + 3^2}$	M1	Use of Pythagoras	
	= 18.248… so 18.2 N (3 s. f.)	A1	Any reasonable accuracy. FT **R** (with 2 non-zero cpts)	
	angle is $180 - \arctan\left(\frac{3}{18}\right)$ = 170.53…°	M1	Allow arctan $\left(\frac{\pm 3}{\pm 18}\right)$ or arctan $\left(\frac{\pm 18}{\pm 3}\right)$	
	so 171° (3 s. f.)	A1	Any reasonable accuracy. FT **R** provided their angle is obtuse but not 180°	4
	total	6		

Q 4		mark		Sub
(i)		B1	All forces present. No extras. Accept *mg*, *w* etc. All labelled with arrows. Accept resolved parts only if clearly additional. Accept no angles	1
(ii)	Resolve parallel to the plane 10 + *T* cos 30 = 4*g* cos 30	M1	All terms present. Must be resolution in at least 1 term. Accept sin $\leftrightarrow$ cos . If resolution in another direction there must be an equation only in *T* with no forces omitted. No extra forces.	
		A1	All correct	
	T = 27.65299… so 27.7 N (3 s. f.)	A1	Any reasonable accuracy	3
(iii)	Resolve perpendicular to the plane *R* + 0.5 *T* = 2*g*	M1	At least one resolution correct . Accept resolution horiz or vert if at least 1 resolution correct. All forces present. No extra forces.	
		A1	Correct. FT *T* if evaluated.	
	R = 5.7735… so 5.77 N (3 s. f.)	A1	Any reasonable accuracy. cao.	3
	total	7		

Q 5		mark		Sub
(i)	$x = 2 \Rightarrow t = 4$ $t = 4 \Rightarrow y = 16 - 1 = 15$	B1 F1	cao FT **their** t and y. Accept 15 **j**	2
(ii)	$x = \frac{1}{2}t$ and $y = t^2 - 1$ Eliminating t gives $y = \left((2x)^2 - 1\right) = 4x^2 - 1$	M1 E1	Attempt at elimination of expressions for x and y in terms of t Accept seeing $(2x)^2 \text{-} 1 = 4x^2 - 1$	2
(iii)	**either** We require $\frac{dy}{dx} = 1$ so $8x = 1$ $x = \frac{1}{8}$ and the point is $\left(\frac{1}{8}, -\frac{15}{16}\right)$ **or** Differentiate to find **v** equate **i** and **j** cpts so $t = \frac{1}{4}$ and the point is $\left(\frac{1}{8}, -\frac{15}{16}\right)$	 M1 B1 A1 M1 M1 A1	 This may be implied Differentiating correctly to obtain $8x$ Equating the **i** and **j** cpts of **their v**	 3
	total	7		

Q 6		mark		Sub
(i)	$2000 = 1000a$ so $a = 2$ so 2 m s^{-2}	B1		
		M1	Use of appropriate *uvast* for t	
	$12.5 = 5 + 2t$ so $t = 3.75$ so 3.75 s	A1	cao	3
(ii)	$2000 - R = 1000 \times 1.4$	M1	N2L. Accept $F = mga$. Accept sign errors. Both forces present. Must use $a = 1.4$	
	$R = 600$ so 600 N (AG)	E1		2
(iii)	$2000 - 600 - S = 1800 \times 0.7$	M1	N2L overall or 2 paired equations. $F = ma$ and use 0.7. Mass must be correct. Allow sign errors and 600 omitted.	
		A1	All correct	
	$S = 140$ so 140 N (AG)	E1	Clearly shown	3
(iv)	$T - 140 = 800 \times 0.7$	M1	N2L on trailer (or car). $F = 800a$ (or $1000a$). Condone missing resistance otherwise all forces present. Condone sign errors.	
		B1	Use of 140 (or 2000 – 600) and 0.7	
	$T = 700$ so 700 N	A1		3
(v)	N2L in direction of motion car and trailer			
	$-600 - 140 - 610 = 1800a$	M1	Use of $F = 1800a$ to find new accn. Condone 2000 included but not T. Allow missing forces.	
		A1	All forces present; no extra ones Allow sign errors.	
	$a = -0.75$	A1	Accept $\pm$. cao.	
	For trailer $T - 140 = -0.75 \times 800$	M1	N2Lwith their a ($\neq 0.7$) on trailer or car. Must have correct mass and forces. Accept sign errors	
	so $T = -460$ so 460	A1	cao. Accept ± 460	
	thrust	F1	Dep on M1. Take tension as +ve unless clear other convention	6
	total	17		

Q 7		mark		Sub
(i)	$u = \sqrt{10^2 + 12^2} = 15.62..$	B1	Accept any accuracy 2 s. f. or better	
	$\theta = \arctan\left(\frac{12}{10}\right) = 50.1944...$ so 50.2 (3s.f.)	M1	Accept $\arctan\left(\frac{10}{12}\right)$ (Or **their** $15.62\cos\theta = 10$ or **their** $15.62\sin\theta = 12$)	
		A1	[FT **their** 15.62 if used] [If θ found first M1 A1 for θ F1 for u] [If B0 M0 SC1 for both $u\cos\theta = 10$ and $u\sin\theta = 12$ seen]	3
(ii)	vert $12t - 0.5\times 10t^2 + 9$	M1	Use of $s = ut + 0.5at^2$, $a = \pm 9.8$ or ± 10 and u = 12 or 15.62.. Condone $-9 = 12t - 0.5\times 10t^2$, condone $y = 9 + 12t - 0.5\times 10t^2$. Condone g.	
		A1	All correct with origin of u = 12 clear; accept 9 omitted	
	$= 12t - 5t^2 + 9$ (AG)	E1	Reason for 9 given. Must be clear unless $y = s_0 + ...$ used.	
	horiz $10t$	B1		4
(iii)	$0 = 12^2 - 20s$	M1	Use of $v^2 = u^2 + 2as$ or equiv with u = 12, v = 0. Condone $u \leftrightarrow v$	
	s = 7.2 so 7.2 m	A1	From CWO. Accept 16.2.	2
(iv)	We require $0 = 12t - 5t^2 + 9$	M1	Use of y equated to 0	
	Solve for t	M1	Attempt to solve a 3 term quadratic	
	the + ve root is 3	A1	Accept no reference to other root. cao.	
	range is 30 m	F1	FT root and **their** x. [If range split up M1 all parts considered; M1 valid method for each part; A1 final phase correct; A1]	4
(v)	Horiz displacement of B: $20\cos 60t = 10t$	B1	Condone unsimplified expression. Award for 20cos60 = 10	
	Comparison with Horiz displacement of A	E1	Comparison clear, must show $10t$ for each or explain.	2
(vi)	vertical height is $20\sin 60t - 0.5\times 10t^2 = 10\sqrt{3}t - 5t^2$ (AG)	A1	Clearly shown. Accept decimal equivalence for $10\sqrt{3}$ (at least 3 s. f.). Accept $-5t^2$ and 20sin60 = $10\sqrt{3}$ not explained.	1
(vii)	Need $10\sqrt{3}t - 5t^2 = 12t - 5t^2 + 9$	M1	Equating the **given** expressions	
	$\Rightarrow t = \frac{9}{10\sqrt{3} - 12}$	A1	Expression for t obtained in any form	
	t = 1.6915… so 1.7 s (2 s. f.) (AG)	E1	Clearly shown. Accept 3 s. f. or better as evidence. Award M1 A1 E0 for 1.7 sub in each ht	3
	total	19		

4761 - Mechanics 1

General Comments

Most candidates seemed to be able to do a substantial amount of the paper with quite a few doing well on every question. There were relatively few candidates who could not make any real progress with any question. Most candidates did well on Q2, 3, 6, and 7. The responses to the two section B questions were especially pleasing with many essentially complete solutions to each. Many candidates had major problems with one or more of Q1, 4 and 5. Perhaps it was the case that these questions somehow did not allow some candidates to show what they knew but there was an impression given that many of them were not familiar with the techniques required.

As always there were many beautifully presented scripts with clear, accurate working and full accounts given of the methods used but many other candidates lost marks because of slips, poor (or no) diagrams and lack of adequate explanation, especially of *given* results that had to be *shown*.

Comments on Individual Questions

1) **The use of an acceleration-time graph**

This question gave a bad start to many candidates as it seems they did not realize that the area under an acceleration – time graph represents change in velocity. These usually scored the first mark (showing that they did understand it was an acceleration – time graph) and could also get the third mark for writing down that $a = 2t$. Otherwise, they mostly either tried to use the constant acceleration results or argued that velocities were connected to gradients so they needed the gradients of the lines on the graphs.

(i) The correct acceleration was found by almost every candidate but even some who realized they should be finding an area miscalculated to get 32 instead of 16 m s^{-1}.

(ii) Answered correctly by the majority of the candidates.

(iii) Even some candidates who realized that the area under the curve represented velocity change thought that the answer was $t = 5$, where the acceleration is greatest, instead of $t = 7$ where the acceleration changes from positive to negative. Some who correctly wrote $t = 7$ gave as their reason 'the acceleration is zero' instead of it noting that it changes sign.

(iv) Only those who understood that the area represented velocity change could score marks here and those that understood did well with few slips.

2) **Kinematics in 1 dimension using calculus**

Very many candidates scored full marks on this question, including some who did not do well overall. There was no pattern to the few errors that were not slips except, of course, for the minority who tried to use the constant acceleration results to find the distance travelled.

3) **Equilibrium of forces given in terms of unit vectors; the magnitude and direction of a vector**

(i) Most candidates found **R** correctly, including the sign. This is pleasing as a sign error has been more common in recent sessions.

(ii) Most candidates obtained the correct magnitude of **R** but only a few realized that its direction is an angle in the second quadrant and so they obtained the wrong angle with **i**.

4) **A heavy block in equilibrium on an inclined plane**

Although quite a few candidates worked through this problem accurately and efficiently, many others scored few marks at all.

(i) Many of the diagrams were poor. Some candidates did not even show four forces, as requested, usually omitting the normal reaction. Many diagrams failed to show the weight and/or the tension in the string to be vertical and others introduced an extra force marked F (for friction?) down the plane. Labelling was often incomplete and in many cases arrows were missed out.

(ii) By no means all the candidates followed the instruction to resolve parallel to the slope. Those that did not do this were not penalized in this case but usually forgot the component of the normal reaction. Many candidates omitted at least one force and/or failed to resolve both the tension and the weight. The resolution attempted was, in many cases wrong.

(iii) All the mistakes seen in part (ii) were seen here also. By far the most common error (seen in many sessions in the past) is to take the normal reaction to be that component of the weight perpendicular to the plane.

5) **The kinematics of a particle moving in 2 dimensions, given in vector form**

Although some fully correct solutions were seen, few candidates managed part (iii) satisfactorily.

(i) Most candidates knew what to do to find t but a surprisingly large minority argued that $0.5t = 2$ implies that $t = 1$.

(ii) Only about half of the candidates seemed to know that they should solve the linear equation for *t* and then eliminate *t* from the quadratic equation. Some who did failed to give enough working to *show* the result.

(iii) Very few candidates realized that direction of the movement is the direction of the velocity; instead, they mostly tried to equate the **i** and **j** components of the position vector. The successful attempts were approximately evenly split between those who differentiated the cartesian equation of the path to find where the gradient of the tangent is 1 and those who differentiated **r** and then found where the **i** and **j** components of **v** are the same.

6) **The motion of a car and a trailer and the force in the tow-bar**

Very many candidates did well on all parts of this question.

(i) Almost all the small number of errors were miscopies or slips

(ii) Most candidates knew what to do but some solutions based on showing all the figures given were consistent with Newton's second law did not properly *show* the given result – some candidates wrote 2000 – 600 = 1000 x 1.4 without any indication of method or comment.

(iii) Again in this part, most candidates knew what to do but some failed to *show* the result as in part (ii). Quite a few candidates analysed the motion of the car and trailer separately and so found the force in the tow-bar requested in part (iv).

(iv) I felt that a higher proportion of the candidates knew what to do than in some recent sessions but there were still attempts that incorporated the weight of the car and trailer and attempts not based on the application of Newton's second law at all. As mentioned above, some candidates had already found the required value because of their approach to part (iii).

(v) I was pleased to see so many complete solutions to this part. Most candidates knew how to find the new acceleration but many made mistakes with the signs of some of the terms. A common error was to omit the resistance of the trailer or the car. Lack of a clear sign convention added to the problems, especially when moving to the use of the new acceleration to find the new force in the tow-bar so that, for instance, a value of acceleration with backwards positive was used with forces signed as if forwards were positive. A minority of candidates do not look to their calculations to decide on whether the force is a tension or a thrust but seem to think the answer is to be found from qualitative arguments.

7 **A projectile problem**

There were many complete solutions to this question and many more almost complete.

(i) Almost every candidate correctly obtained the required values

(ii) Most candidates knew what to do but many gave too little explanation to *show* a given result. Commonly, the + 9 of the expression for the vertical height appeared with no or inadequate explanation. Many candidates forgot to give an expression for the horizontal distance travelled.

(iii) Most candidates knew what to do, although many gave the height above the ground – this was not penalized in this case. As always when using the result $v^2 = u^2 + 2as$, there were some sign errors seen. Quite a few candidates rather inefficiently used a method requiring finding the time to the greatest height first.

(iv) A pleasing number of candidates did this part very well. Those who tried to consider the flight in sections tended to forget the section from the height of projection to the ground or failed to find correctly the time for this part of the flight.

(v) Many candidates did not give a complete argument and so lost one of the marks.

(vi) This was usually done well.

(vii) There were many good answers to this part. Again, marks were lost because the given result was not completely established. A quite common and surprising error in the light of parts (v) and (vi) was simply to show that the horizontal displacements were the same at the given time.